Instant Radiosity

by

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Outline of Talk

- Global Illumination and Radiosity
- Basic Techniques:
  - Quasi-Monte Carlo Integration
  - Quasi-Random Walk
  - Accumulation Buffer
- The Instant Radiosity Algorithm
- Extensions, Conclusion and Future Research

The Global Illumination Problem

- Given by (S, f_r, L_e, Ψ)
  - reflectance distribution $f_r(\vec{\omega}_i, y, \vec{\omega}_r)$ on the surface S
  - source radiance $L_e(y, \vec{\omega}_r)$ from $y \in S$ into direction $\vec{\omega}_r$
  - flux detector $\Psi(y, \vec{\omega})$

- Seeking for

$$\langle L, \Psi \rangle := \int_S \int_\Omega L(y, \vec{\omega}) \Psi(y, \vec{\omega}) \cos\theta d\omega dy$$

where

$$L(y, \vec{\omega}_r) = L_e(y, \vec{\omega}_r) + \underbrace{\int_\Omega f_r(\vec{\omega}_i, y, \vec{\omega}_r) L(h(y, \vec{\omega}_i), -\vec{\omega}_i) \cos\theta_i d\omega_i}_{(T_{f_r} L)(y, \vec{\omega}_r)}$$

  - first point $h(y, \vec{\omega}_i) \in S$ hit by the ray from y into direction $\vec{\omega}_i$

- Special case: Diffuse environments (radiosity setting)

$$f_r = f_d(y) := \frac{\rho_d(y)}{\pi}$$

State of the Art

- Galerkin method with kernel and solution discretization

$$L_i = L_{e,i} + \rho_i \sum_{j=1}^{K} L_j f_{ij}$$

- Random walk approach with only solution discretization

$$L(y) \approx \sum_{k=1}^{K} \langle L, \Psi_k \rangle \Psi_k(y)$$

- Bidirectional path tracing

$$\langle L, \Psi_P \rangle = \sum_{j=0}^{\infty} \sum_{n=0}^{j} w_{jn} \langle T_{f_r}^{n} L_e, T_{f_r}^{*(j-n)} \Psi_P \rangle$$

The new Approach

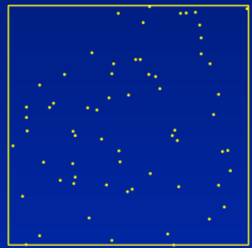
- Goals:
  - very fast and simple algorithm
  - avoid memory overhead introduced by discretizations
  - allow textures, cyclic scene graphs, dynamic environments
- Means:
  - low discrepancy sampling (quasi-Monte Carlo integration)
  - standard graphics hardware and rendering pipelines

Quasi-Monte Carlo Integration

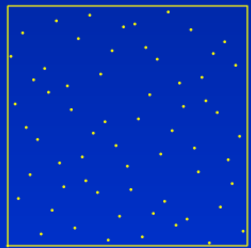
- Uniformly distributed points $P_N = \{x_0, \ldots, x_{N-1}\} \subset [0,1)^s$

$$\int_{[0,1)^s} f(x)dx \approx \frac{1}{N} \sum_{i=0}^{N-1} f(x_i)$$

- Convergence rate
  - for random sampling $\mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$



  - for low discrepancy sampling $\mathcal{O}\left(\frac{\log^s N}{N}\right)$



Low Discrepancy Sample Points

- Radical inversion of i in base b

$$\Phi_b(i) := \sum_{j=0}^{\infty} a_j(i)\, b^{-j-1} \Leftrightarrow i = \sum_{j=0}^{\infty} a_j(i)\, b^j$$

- Halton sequence

$$x_i = \left(\Phi_{b_1}(i), \ldots, \Phi_{b_s}(i)\right)$$

- Hammersley point set

$$x_i = \left(\frac{i}{N}, \Phi_{b_1}(i), \ldots, \Phi_{b_{s-1}}(i)\right)$$

The Quasi-Random Walk

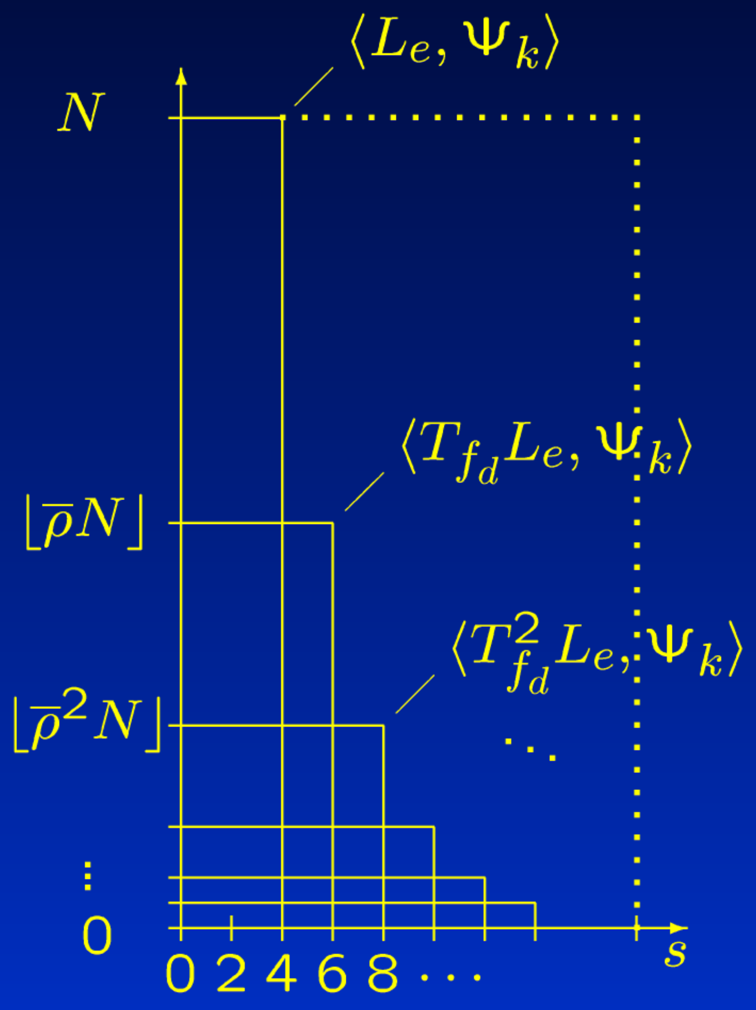
- Projection onto $(\Psi_k)_{k=1}^{K}$ spanning $S =$ supp $\sum_{k=1}^{K} \Psi_k$

$$L(y) \approx \sum_{k=1}^{K} \langle L, \Psi_k \rangle \Psi_k(y)$$

- $\bar{\rho} = \frac{\sum_{k=1}^{K} |A_k| \rho_k^d}{\sum_{k=1}^{K} |A_k|} \approx \|T_{f_d}\| < 1 \Rightarrow$ convergent Neumann series

$$\Rightarrow \langle L, \Psi_k \rangle = \underbrace{\langle L_e, \Psi_k \rangle}_{N} + \underbrace{\langle T_{f_d} L_e, \Psi_k \rangle}_{\lfloor \bar{\rho} N \rfloor} + \underbrace{\langle T_{f_d}^2 L_e, \Psi_k \rangle}_{\lfloor \bar{\rho}^2 N \rfloor} + \cdots$$

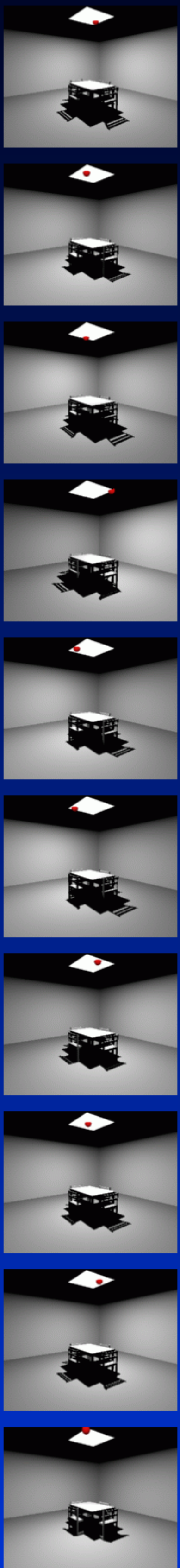
```
Start = End = N; Reflections = 0;
while(End > 0)
{
   Start *= ρ̄;
   for(i = (int) Start; i < End; i++)
   {
      Sample position y
      for(j = 0; j <= Reflections; j++)
      {
         Record contribution to Ψ_k in y
         Sample direction ω⃗
         y = h(y,ω⃗);
         Attenuate particle
      }
   }
   Reflections++;
   End = (int) Start;
}
```

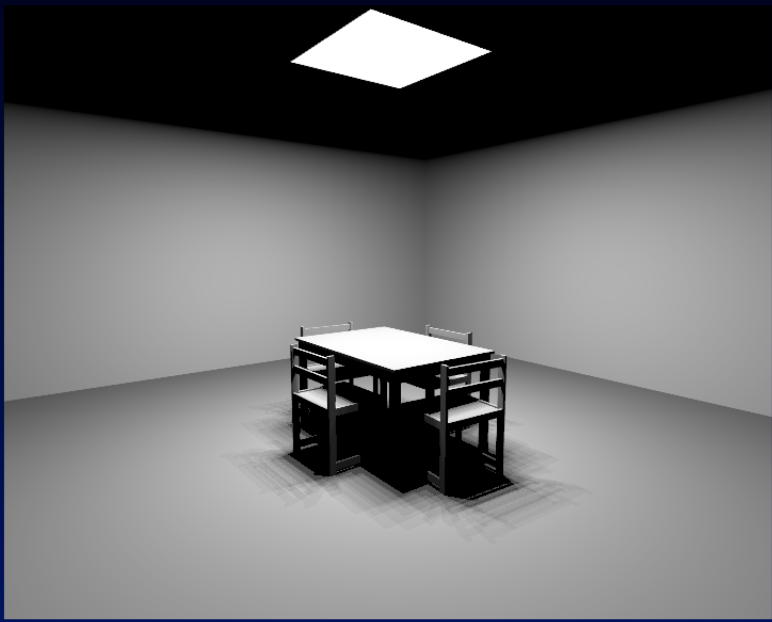


The Accumulation Buffer

- Direct illumination by extended light sources

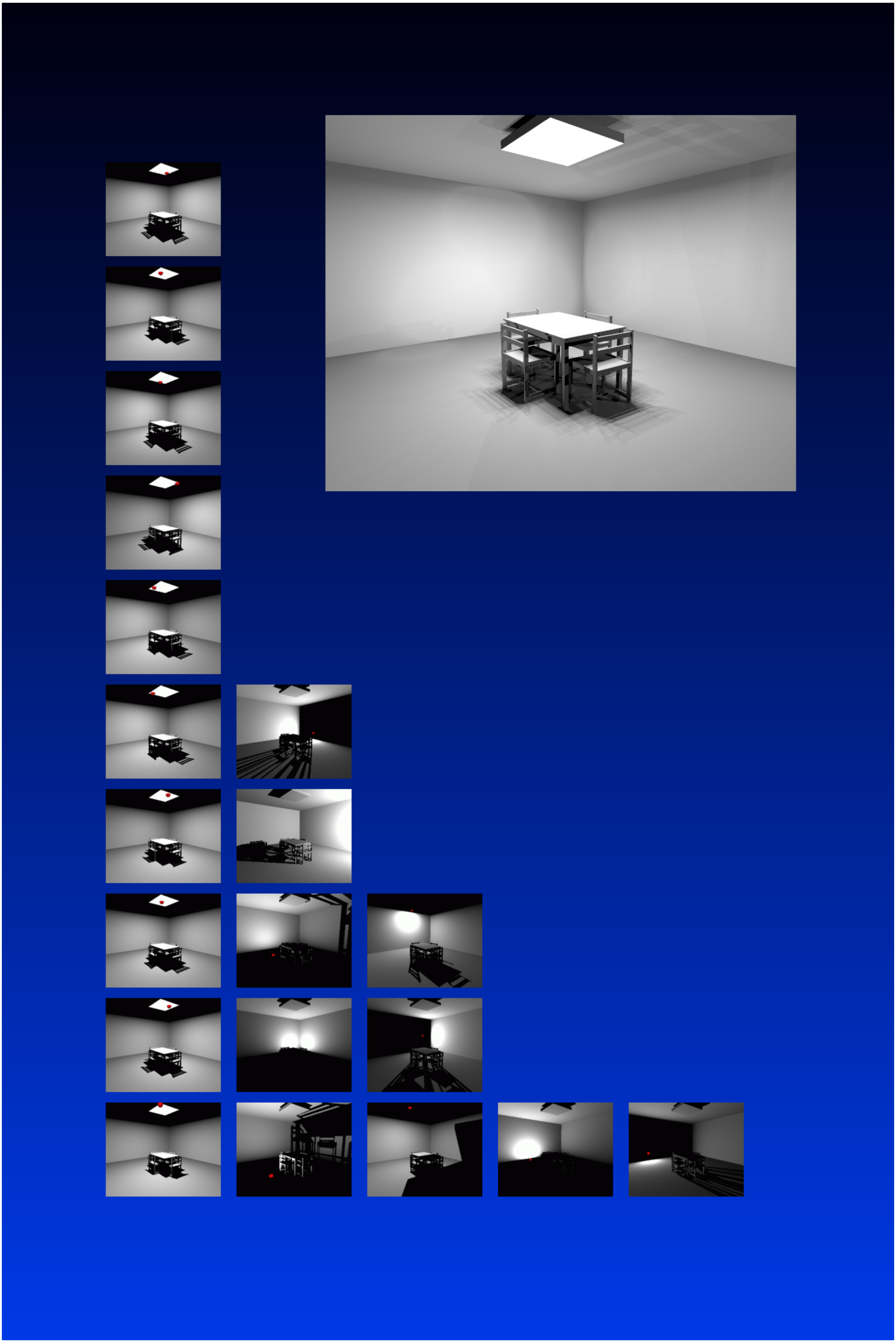
```
for(int i = 0; i < N; i++)
{
    Sample position y
    L = Le(y) * supp Le;
    glRenderShadowedScene(L, y);
    glAccum(GL_ACCUM, 1/N);
}
glAccum(GL_RETURN, 1.0);
```





The Instant Radiosity Algorithm

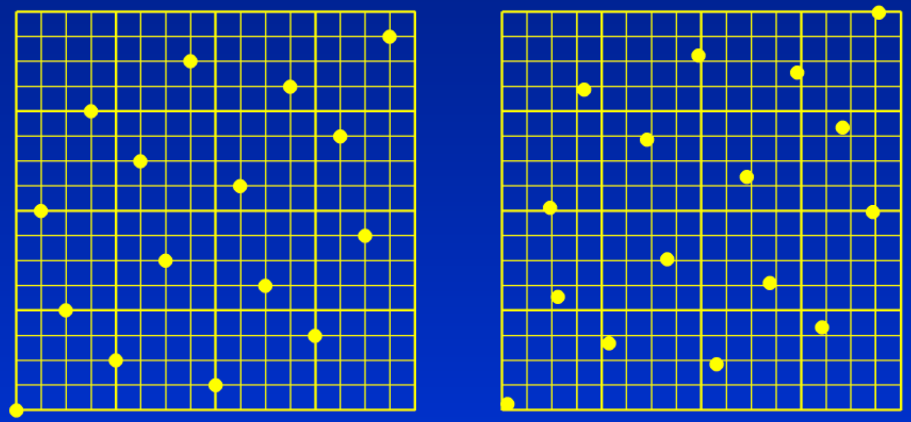
```
Start = End = N; Reflections = 0;
while(End > 0)
{
    Start *= ρ̄;
    for(int i = (int) Start; i < End; i++)
    {
        Sample position y
        L = L_e(y) * supp L_e;
        for(int j = 0; j <= Reflections; j++)
        {
            glRenderShadowedScene(N/⌊ρ̄^j N⌋ L, y);
            glAccum(GL_ACCUM, 1/N);
            Sample direction ω⃗
            y = h(y, ω⃗);
            L *= f_d(y);
        }
    }
    Reflections++;End = (int) Start;
}
glAccum(GL_RETURN, 1.0);
```



Conference room Image

Extensions

- Specular effects
  - last reflection: Full BRDF by hardware
  - virtual light sources
  - multipass techniques
- Jittered low discrepancy sampling



Radiosity in Dynamic Environments

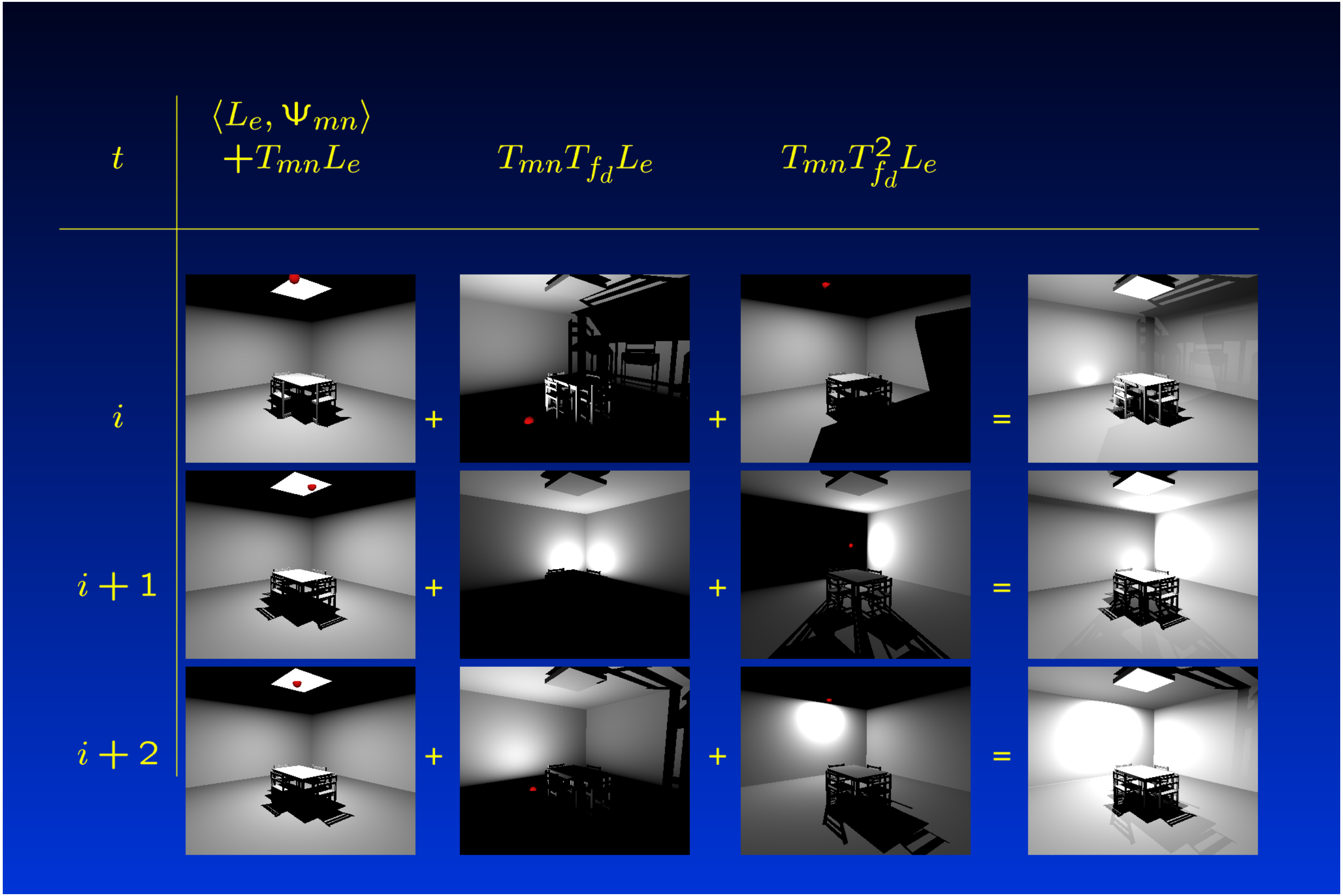
- Fixed length quasi-random walk

$$l_{max} := \left\lfloor -\frac{\log N}{\log \overline{\rho}} \right\rfloor$$

  - frame rate divided by $l_{max} \Rightarrow$ (close to) realtime

- Display with motion blur

  - superimpose the last N composite frames with weight $\frac{1}{N}$



t
$\langle L_e, \Psi_{mn} \rangle + T_{mn} L_e$
$T_{mn} T_{f_d} L_e$
$T_{mn} T_{f_d}^2 L_e$
i
$i+1$
$i+2$
+
+
=

Conclusion

- Fast and simple global illumination algorithm
  - quasi-Monte Carlo integration
  - existing graphics hardware
- Compatible with standard graphics APIs

Future Research

- Hardware implementation
  - shadow maps
  - specular effects
- Volume scattering
- Adjoint transport operator
- Dynamic environments (AR and VR)